THEORETICAL INVESTIGATIONS OF THE RESISTIVITY - SATURATION RELATIONSHIP IN POROUS MATERIAL

BY

FRANK JOHN REH

Thesis
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Theoretical Investigations of the
Resistivity-Saturation Relationship in Porous Material

By

Frank John Reh

B.S. (United States Naval Academy) 1942

THESIS

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in

Petroleum Engineering

in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA

Approved:

O. J. M. Smith ....

W. H. Somerton ....

J. A. Putnam ....

Committee in Charge

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May 15, 1950

Berkeley, California.

The Faculty of the Department of Engineering,
University of California,
Berkeley, 4, California.

Gentlemen,

In accordance with the regulations of the Graduate Division of the University of California, I submit herewith a thesis entitled "Theoretical Investigations of the Resistivity-Saturation Relationship in Porous Materials".

This is in partial satisfaction of the requirements for the Degree of Master of Science with specialization in Petroleum Engineering at the University of California.

ACKNOWLEDGEMENT

The writer wishes to acknowledge the assistance received from many sources in the completion of this work.

Initially it is desired to acknowledge the indebtedness to the United States Naval Service for placing the writer in a position where such a study was possible. It is to be noted, however, that the ideas and opinions expressed herein are those of the author and do not necessarily represent those of the Navy Department of Naval Service.

The advice, criticisms, and many helpful suggestions given by the committee in charge of this work, Professor W. H. Somerton, Professor J. A. Putnam, and Professor C. J. M. Smith, were invaluable.

Finally the help given by R. W. Voss, R. W. Ravenscroft, and S. P. Miller in the many ideas they presented, did much to help clarify many detailed aspects of the analysis.

THEORETICAL INVESTIGATIONS OF THE RESISTIVITY-SATURATION RELATIONSHIPS IN POROUS MATERIALS

ABSTRACT

In this paper it has been shown that the similarity existing between electrical and hydraulic flow leads to a relationship between electrical conductivity and hydraulic permeability. Expressed in terms of relative resistivity and relative permeability to the wetting phase, the relative permeability is equal to the relative resistivity times the square of the conducting fluid saturation.

Analysis of the data of previous investigators shows that for clean unconsolidated sands the relative permeability is equal to the square of the relative resistivity. For consolidated sands the data are not as definitive, but the indications are that the relative permeability will be less than the square of the relative resistivity.

INTRODUCTION

Since the time of its inception the electrical log has been used primarily to correlate structures and geology in local areas, and to indicate qualitatively a given stratum's fluid content. It is readily apparent that quantitative relationships between fluid content and the data available from the electric log would increase the value of the latter many times.

The problem here is two-fold. There must be established a quantitative relationships between the brine saturation of a porous matrix and its resistivity at this saturation. Further, the relationship of the

true resistivity to that obtained with the log, complicated as it is by many side effects, must be established. This problem is being studied primarily by the oil well servicing companies, and although the answer is not complete, satisfactory empirical relationships have been worked out in the form of curves. (1) The problem of the quantitative saturation-resistivity relationship, with which this paper will concern itself, ahs been primarily one of the laboratory and has been attacked by many investigators. (2)(3)(4)(5)(6)(7)

The first definitive work to extend the usefulness of the electrical log in a quantitative sense was done by Archie. (2) He proposed a method for determining the oil or water saturation of a reservoir bed from a measure of its resistivity as compared to the resistivity of the brine with which it was partially saturated.

From an extensive series of data Archie postulated two empirical relations:

$$F = f^{-m} \qquad (1)$$

$$S = (R_0/R)^{1/n} = (FR_w/R)^{1/n} \qquad (2)$$

where F is a "formation resistivity factor" defined as the resistivity of the saturated matrix (R_0) divided by the resistivity of the brine with which it is saturated (R_w)
f is the fractional porosity,
m is an exponent termed the cementation factor,
S is the fractional brine saturation,
R is the resistivity of the partially saturated matrix, and
n is the saturation exponent, usually taken as 2.

This work opened the field for subsequent investigators to establish a more precise relationship between the brine saturation of a porous matrix and the resultant resistivity. In the main these efforts have been directed

toward establishing a value for "n" that will result in more consistently correct evaluation of the saturation, and toward showing that the resistivity is a function of the method of saturation and of the fluid distribution.

Certainly studies of the variation of resistivity with distribution are very necessary since the distribution of the fluids will probably effect the resistivity. When the medium is wet by the conducting phase the effect is usually small, but marked variations have been shown to occur in some cases when the sand is oil wet.(8)

However, it must be remembered that the data obtained have been applied to an expression which was put forth initially as an approximate relationship without a solid theoretical justification. To overcome this difficulty, the problem was attacked from a fundamental theoretical standpoint. Idealized pore spaces were broken up into incremental elements and the resistivity relationships were analyzed. By this means the relationship between resistivity and saturation was determined for simple idealized pore spaces.

This approach to the problem showed that even in these simple cases the relationships were very complex, and that no simple, general solution of the relationship was available from the geometry of the system alone. Another independent parameter was required. The relationships obtained and their derivations are contained in Appendix I.

CONCEPT OF RELATIVE RESISTIVITY

If the laws of electrical flow are considered, it will be noted

that for a constant electrical voltage and resistivity the amount of current flow per unit of length will be a function of the area available to flow. For hydraulic flow through a capillary tube, for a constant pressure gradient, the rate of fluid flow will be proportional to the radius of the tube raised to the fourth power. This was indicated by Wyllis and Rose(9) when they called attention to "a fundamental difference between viscous resistivity to fluid flow and electrical resistivity. The former effect depends principally on the pore radius to the fourth power (r^4) and the latter to (sic) the radius squared (r^2)".

With this concept in mind the relationship between electrical conductivity and hydraulic permeability will be developed as follows:

Consider a porous matrix saturated with a brine of resistivity R_w. The resistance offered to electrical flow is given by the relation

$$Res = (d/na)R_w \qquad (3)$$

where d is the average length of the pore channels,

n is the number of pore channels, and

a is the average cross sectional area of the pores.

The resistivity of the saturated matrix, R_o, is given by

$$R_o = R_w(d/na)(A/L) = (d/L)(A/na)R_w \qquad (4)$$

where L is the length of the matrix in the direction of electrical flow, and

A is the cross sectional area of the matrix.

If the fractional porosity, f, is written in termes of the geometry of the system

$$f = \frac{\text{void volume}}{\text{bulk volume}} = \frac{(d)(na)}{(L)(A)} \qquad (5)$$

Then

$$(A/na) = (d/L)/f \qquad (6)$$

From which

$$R_o/R_w = (d/L)^2/f \qquad (7)$$

where $(d/L)^2 = T$, is termed the tortuosity of the system.

Rewriting equation (7)

$$R_o/R_w = T/f \qquad (7a)$$

which is at variance to the relationship $R_o/R_w = T^{1/2}/f$ developed by Wyllie and Rose(9) as a definition of formation factor because they based their development on the relation $f = na/A$, which is true only for the capillary tube where $d/L = 1$.

When the electrical and hydraulic flow paths are the same, when no electrically conductive solids are present the permeability may also be expressed as a function of the tortuosity. Rose and Bruce(10) express the hydraulic permeability of a porous matrix by the following relation:

$$K = (fm^2/k_o)(1/T) \qquad (8)$$

in which K is the permeability,
m is the mean hydraulic radius $= f/A_1$,
A_1 is the surface area per unit volume of the matrix forming the pore channels,
k_o is a constant of streamline motion derived from the Kozeny equation, and
f is the fractional porosity.

This may be written as

$$K = (f^3t)/(A_1{}^2k_o) \qquad (9)$$

where $t = 1/T$

For any saturation of the wetting phase the effective porosity of this phase will be equal to the product of the porosity at complete

saturation times the fractional saturation.

$$f_e = Sf \qquad (10)$$

Then for any saturation in which S is less than 1.0

$$K_1 = (f_e{}^3 t)/(A_1{}^2 k_0) \qquad (11)$$

From equation (7a)

$$t = (R_w/R_1)/f_e \qquad (7b)$$

Then

$$\begin{aligned} K_1 &= (f_e{}^2/A_1{}^2 k_0)(R_w/R_1) \\ &= (f^2 S^2/A_1{}^2 k_0)(R_w/R_1) \qquad (12) \end{aligned}$$

Writing permeability at $S = 1.0$ as the homogeneous fluid permeability K, and the permeability at S 1.0 as K_w, and similarily the resistivity at $S = 1.0$ as R_0, and at S 1.0 as R, the ratio K_w/K becomes:

$$\frac{K_w}{K} = \frac{(f^2 S^2/A_1{}^2 k_0)(R_w/R)}{(f^2/A_1{}^2 k_0)(R_w/R_0)}$$

or

$$K_w/K = S^2(R_0/R) \qquad (13)$$

In which K_w/K is the relative permeability to the wetting phase as currently defined in the literature, and R_0/R is a new concept termed the relative resistivity.

ANALYSIS OF DATA

From equation (13) it may be seen that the relative permeability and relative resistivity are both functions of the wetting phase saturation.

Referring to the data of Wyckoff and Botset(11), values of relative permeability to the wetting phase for a clean unconsolidated sand result in a linear relation with saturation on logarithmic coordinates. Figure (5) of the paper by Dunlap and his co-workers(7) is a plot of the resistance of a clean unconsolidated sand against saturation. From these data the quantity $(R_0/R)S^2$ may be calculated. This function is then plotted on logarithmic coordinates against saturation and a straight line may be drawn through Dunlap's experimental data plotted in this manner. For the resistivity function in the region where $S < 0.3$ the fit is excellent. Above this region the experimental points are spread, possibly because of non-uniform saturation. Because no information is available on the gradient of such possible saturation variation, the line represents an "eyeball" average of the data.

These two sets of data are shown on Figure (1). The vertical axes represent the right and left sides of equation (13), and the horizontal axis represents the fractional wetting phase saturation.

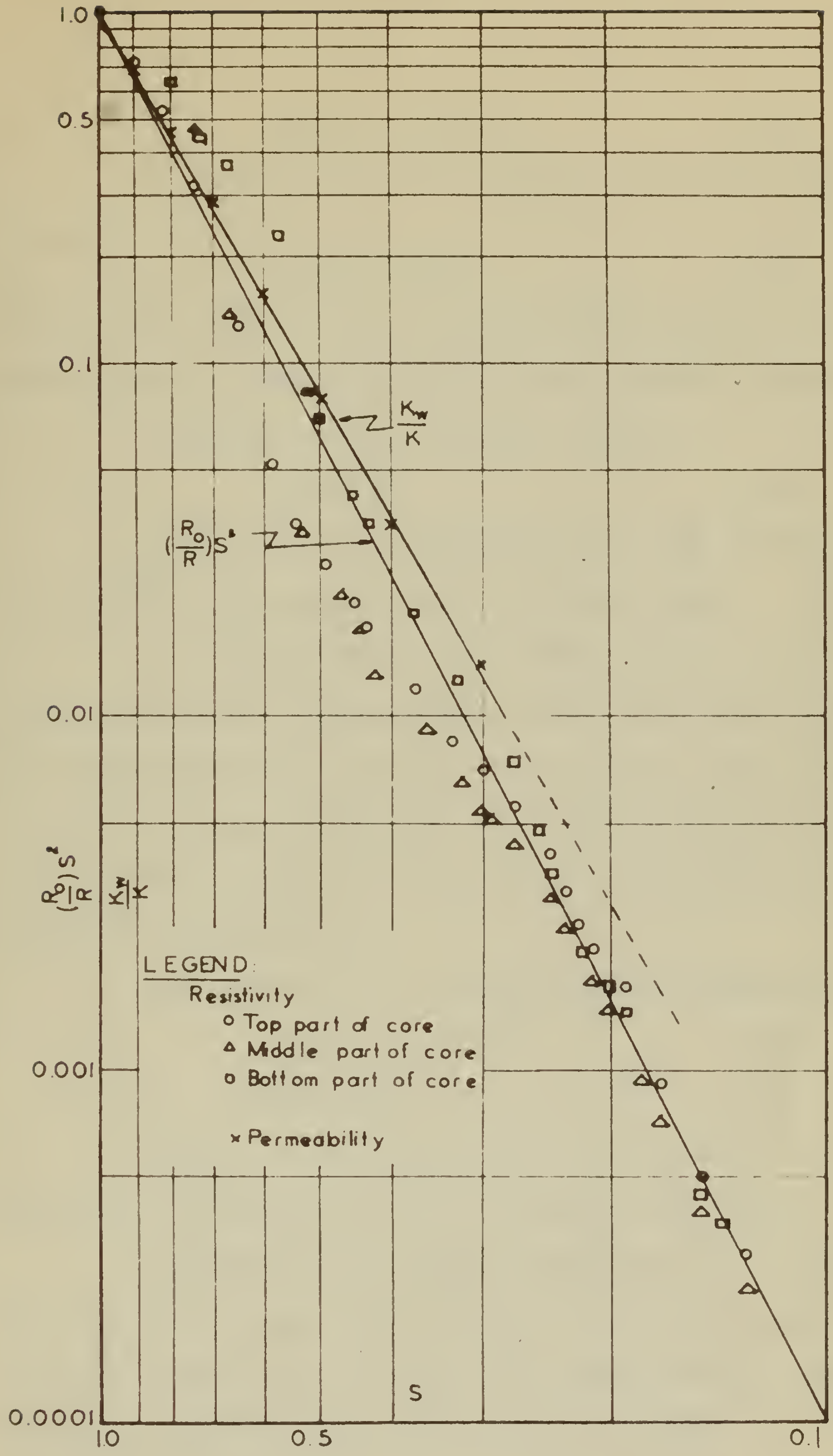
Despite the fact that over ten years seperate the times of obtaining these two sets of data, and despite the fact that the properties of different sands were being investigated, close agreement is obtained between the two lines. Thus it can be said that the function of saturation involved is the same for both parameters.

Thus the equations of these two lines are

$$S^{u-2} = R_0/R \qquad (14a)$$

$$S^{u} = K_w/K \qquad (14b)$$

where u is the slope of the lines.



UNCONSOLIDATED SANDS

RELATIONSHIP BETWEEN RESISTIVITY, PERMEABILITY, AND SATURATION

Relative Resistivity Data After Dunlap (1)

Relative Permeability Data after Wyckoff & Botset (11)

FIGURE I.

Then

$$K_w/K = (R_0/R)^{u/(u-2)} \qquad (15)$$

The slope of the lines in Figure (1) is equal to 4. Equation (15) then becomes

$$K_w/K = (R_0/R)^2 \qquad (16)$$

or for clean, unconsolidated sands the relative permeability to the wetting phase is equal to the square of the relative resistivity.

Going back to equation (14a), for $u = 4$

$$R_0/R = S^2 \qquad (17)$$

Thus for clean, unconsolidated sands, the relation postulated by Archie(2), using an exponent of 2, is quite valid.

For consolidated sands the data available in the literature are not definitive. Using the data of Morse and his co-workers(12), for consolidated sands, the right and left sides of equation (13) are again plotted against saturation on logarithmic coordinates, Figure (2). The lines thus obtained are not as good as those of Figure (1), but yield a slope of approximately 6.0.

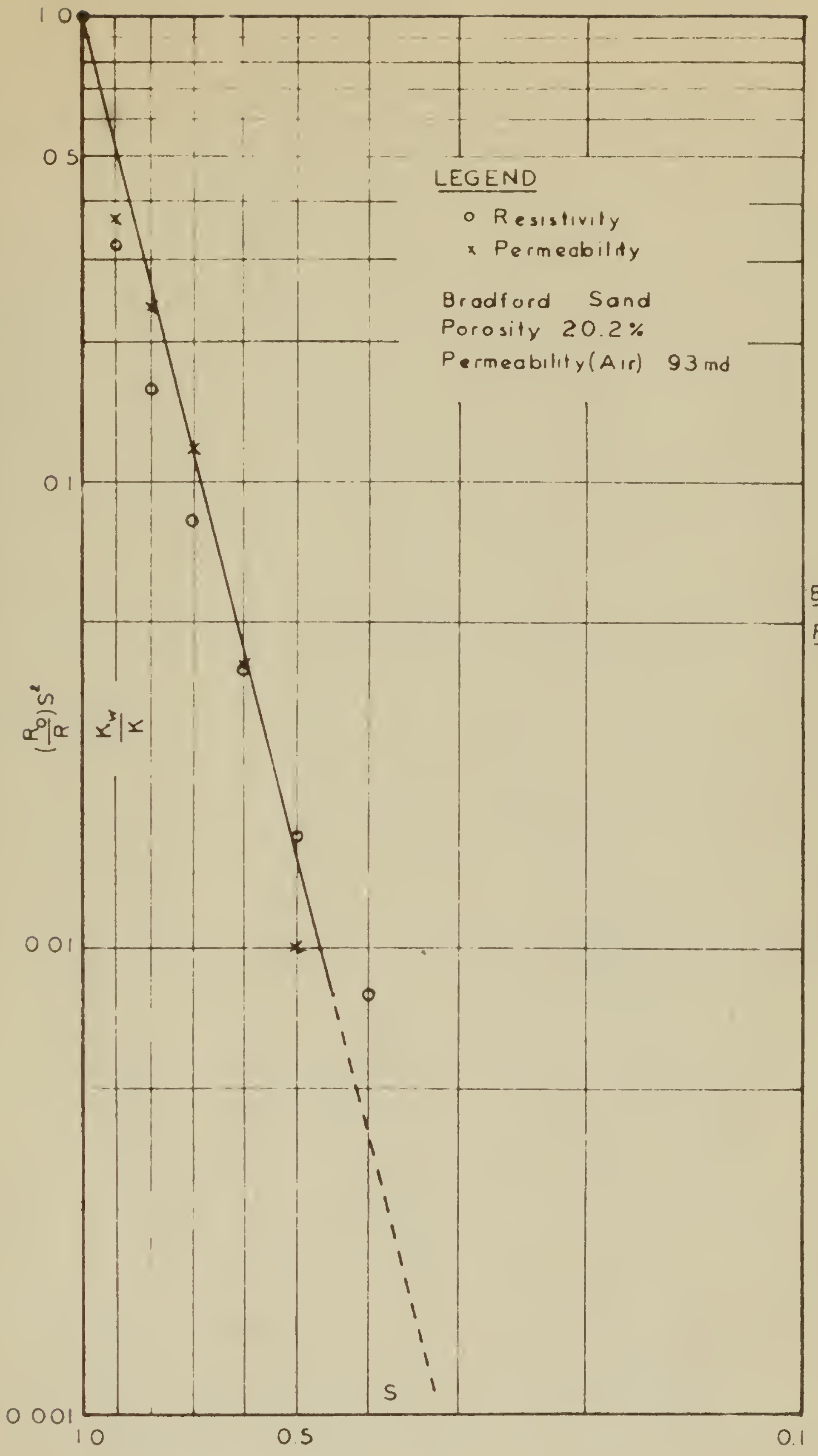
Then for consolidated sands approximate relations may be written

$$K_w/K = (R_0/R)^{1.5} \qquad (18)$$

$$S^4 = R_0/R \qquad (19)$$

Because of the dearth of data for consolidated sands equations (18) and (19) are presented only as qualitative relations.

Considering the case of the capillary tube, it is known that the resistivity varies linearly with the saturation. From this the function $(R_0/R)S^2$ may be calculated and plotted against saturation. Figure (3) shows the slope of this line to be three.

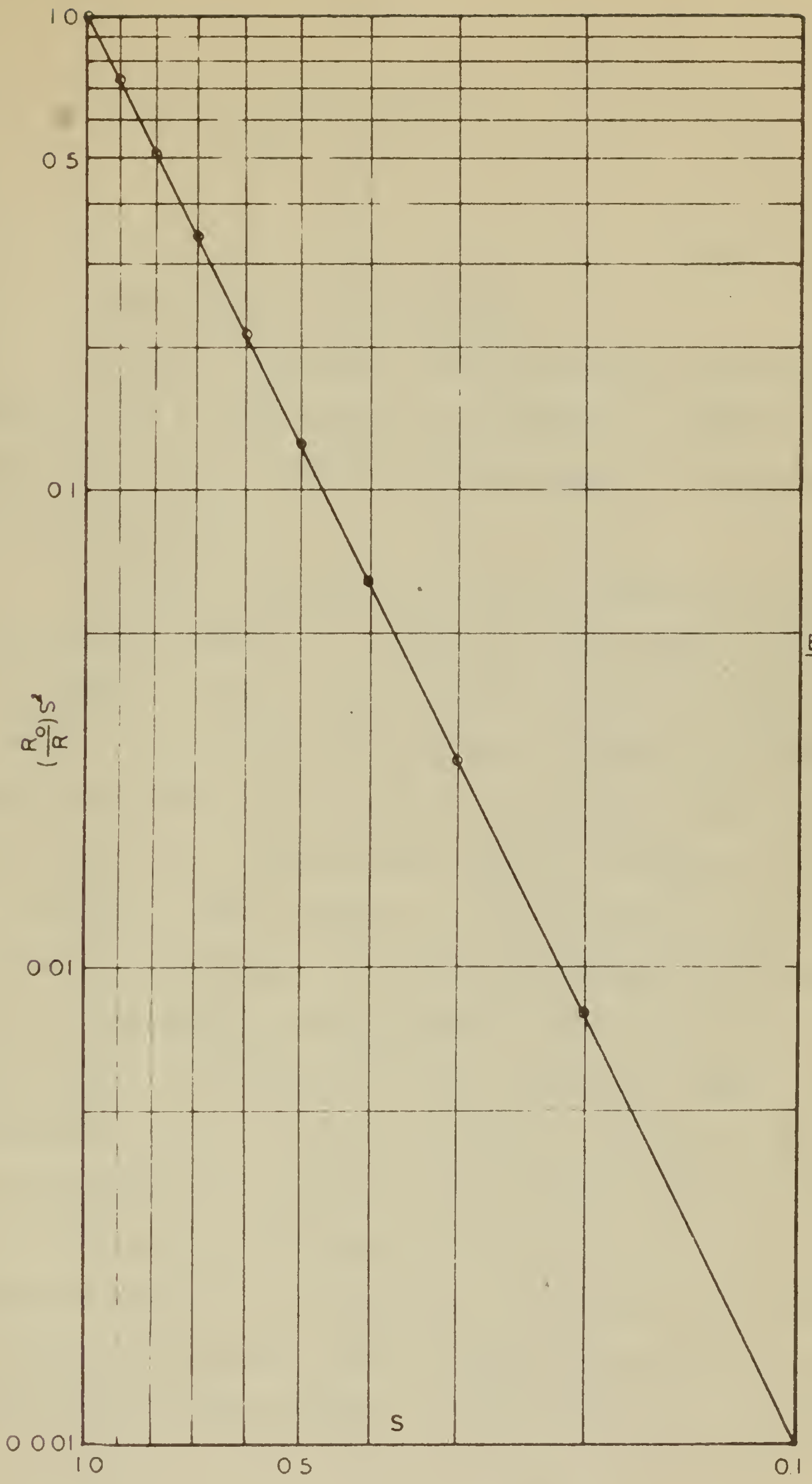


CONSOLIDATED SANDS

RELATIONSHIP BETWEEN RESISTIVITY, PERMEABILITY, AND SATURATION

Data After Morse (12)

FIGURE 2



CAPILLARY
TUBE

RELATIONSHIP
BETWEEN RESISTIVITY
AND SATURATION

FIGURE 3.

Thus for a capillary tube

$$S^{u-2} = R_0/R$$

or

$$S = R_0/R \qquad (20)$$

which relation is indeed valid.

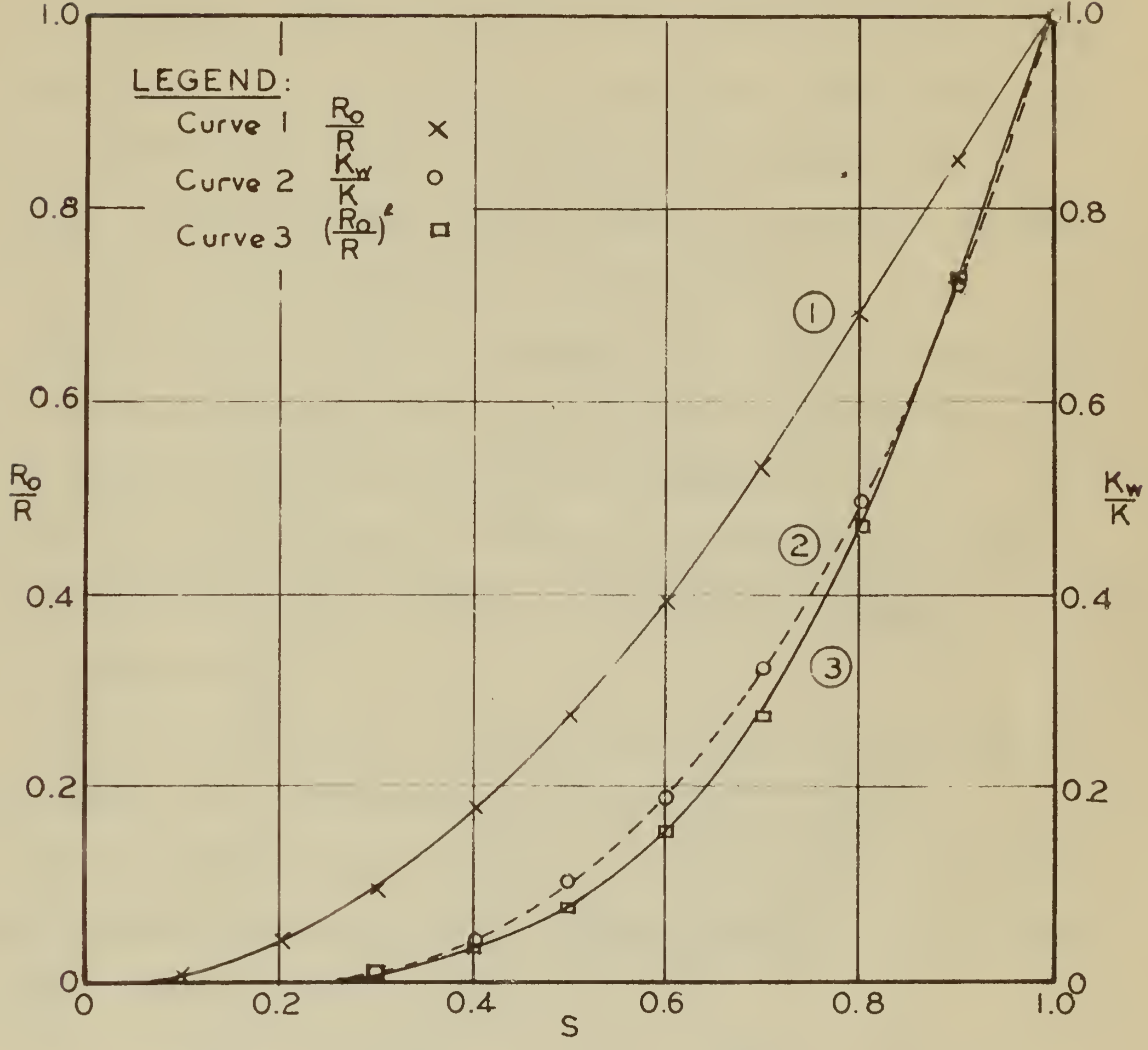
It must be remembered, however, that these relationships are applicable only when the hydraulic and electric flow paths are the same, or when there are no conducting solids to conduct the electricity to the exclusion of fluid flow.

Additional data are available in the literature in the work of previous investigators(14)(15). These data along with those previously used are shown in tabular form in Appendix II and the source of each is indicated. In all cases, in order to obtain relative permeability and relative resistivity values at the same saturations, these values were taken from the curves drawn by these previous investigators rather than the experimental points themselves. These data are grouped for unconsolidated sands, consolidated sands, and synthetic consolidated cores.

Using the arithmetic average of the four sets of data given in Tables I through IV (Appendix II), three curves were plotted on rectangular coordinates, (Figure 4.). The first curve is a plot of relative resistivity versus saturation, the second is a plot of relative permeability to the wetting phase versus saturation, and the third is a plot of relative resistivity squared versus saturation. The spread of the relative resistivity data used to obtain curve 1 of Figure (4), is shown by Figure (8), Appendix II. The close agreement of curves 2 and 3 supports the

AVERAGE VALUES OF RELATIVE RESISTIVITY AND RELATIVE PERMEABILITY FOR UNCONSOLIDATED SANDS

(TABLE V)



NOTE: Curves 2 and 3 show relationship $\frac{K_w}{K} = (\frac{R_o}{R})^2$; Equation (16)

FIGURE 4.

relationship given by equation (16).

Referring to the data given by Morse(12) for a Bradford sand, (Table VI, Appendix II), curves similar to those drawn for the unconsolidated sand are shown for this one set of data on a consolidated sand, Figure (5). In addition a fourth curve is drawn to represent the right hand side of equation (13). The fair agreement between curves 2 and 4 tends to support the relation given by equation (18) for this set of data.

Finally using the data for a synthetic core given by Morse (12) the three curves shown in Figure (4) are plotted for a synthetic core. (Table VII, Appendix). Figure (6) shows these relationships but the data available for this case were so scant that no definite conclusions may be drawn therefrom, except that this set of data gives excellent support to equation (16).

If all the data and the fundamental ideas developed above are considered together in a more general sense an additional possible aspect of the relationship becomes apparent. It may be noted that for the cases of the unconsolidated sands and the synthetic core the data available are in agreement with the relationships

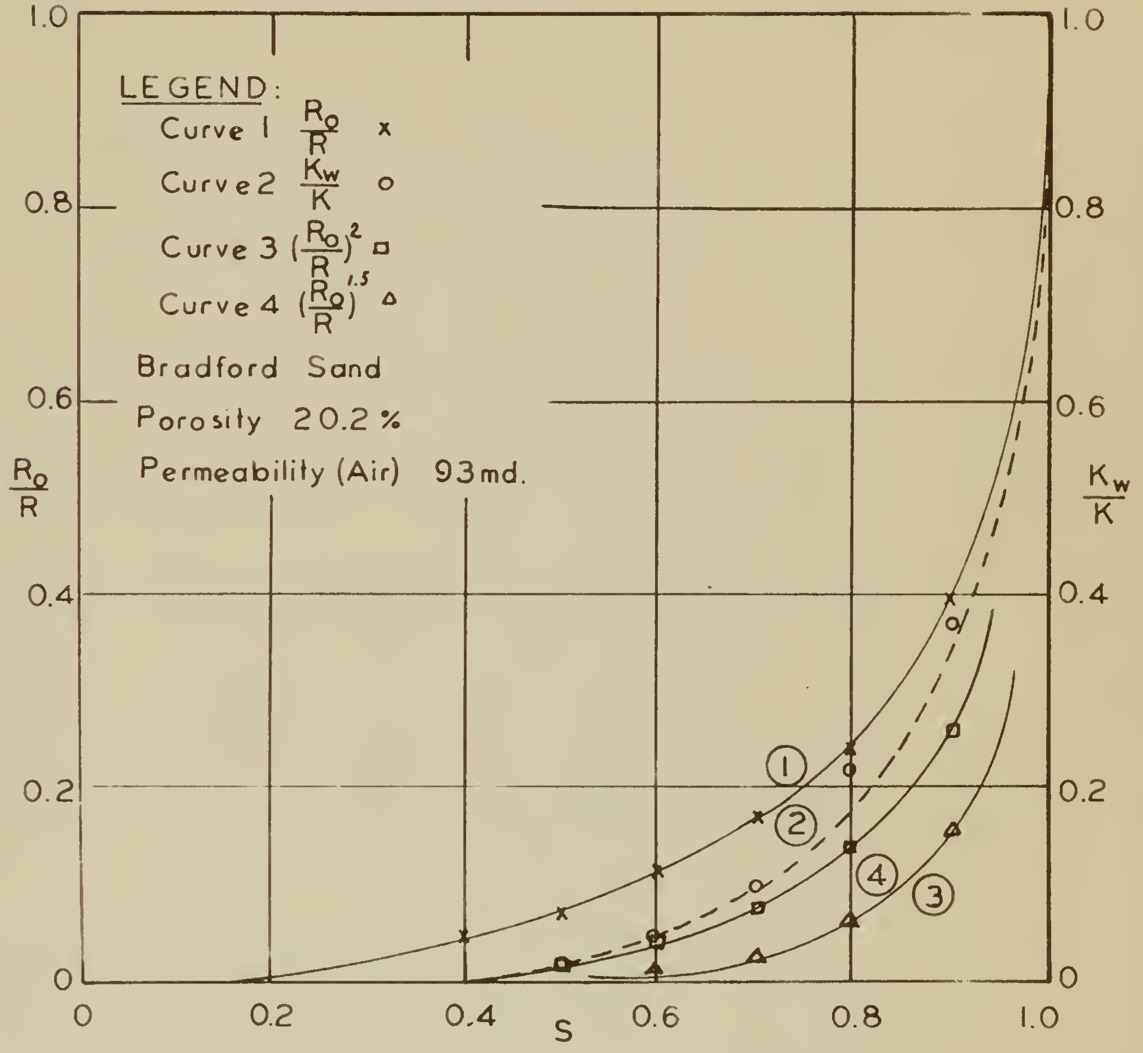
$$K_w/K = (R_o/R)^2 \qquad \text{and}$$

$$R_o/R = S^2$$

The main point of physical similarity between the unconsolidated sands and the synthetic core is that both are, to all practical purposes, isotropic. There are no bedding planes and no stratification. With this in mind, note that for the case of the capillary tube the relation between relative

RELATIONSHIP BETWEEN RELATIVE RESISTIVITY AND RELATIVE PERMEABILITY FOR A CONSOLIDATED SAND

Data After Morse (12)

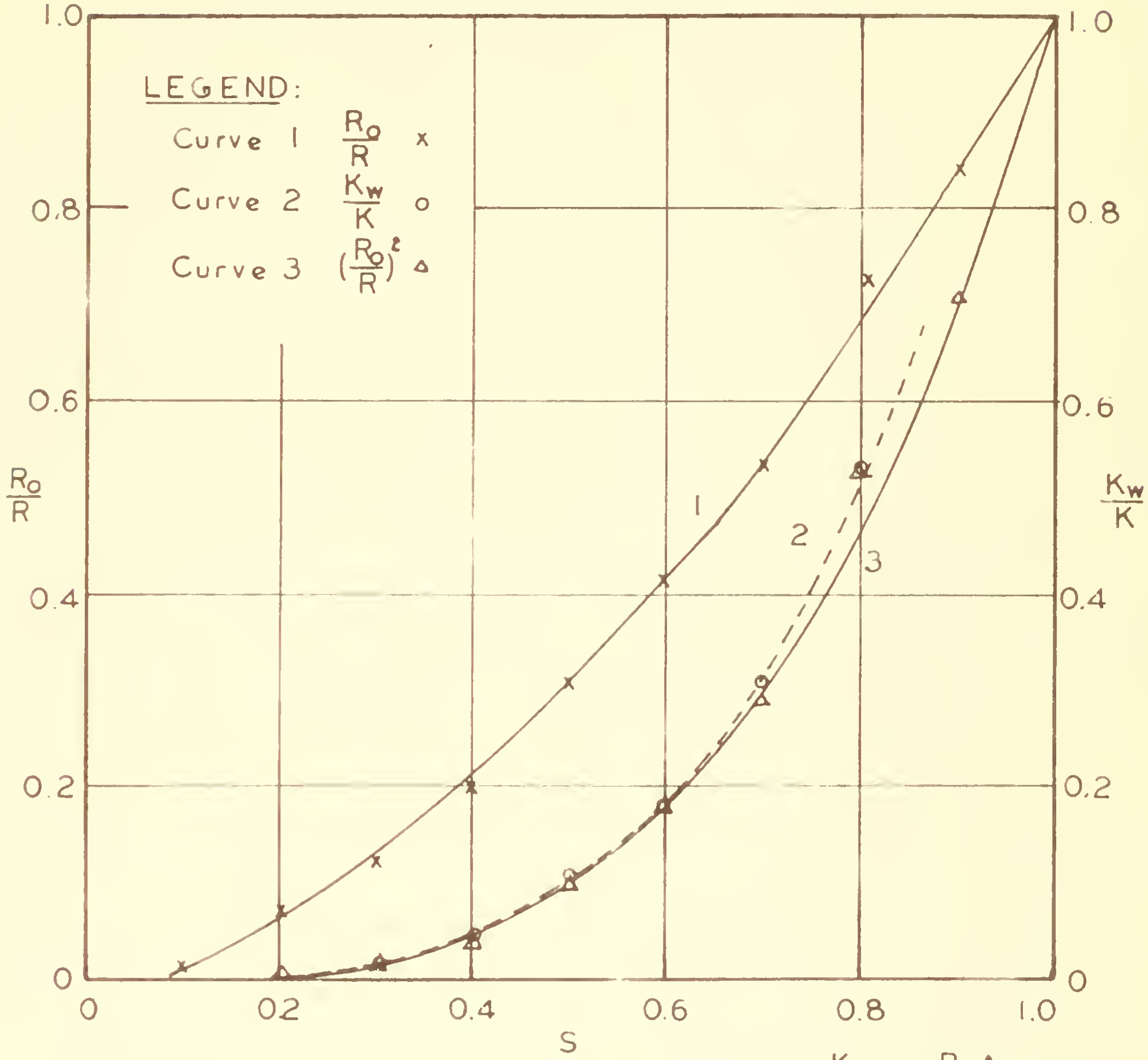


NOTE: Curves 2 and 4 show relationship $\frac{K_w}{K} = (\frac{R_o}{R})^{1.5}$; Equation (18).

FIGURE 5.

RELATIONSHIP BETWEEN RELATIVE RESISTIVITY AND RELATIVE PERMEABILITY FOR A CONSOLIDATED SYNTHETIC CORE

Data After Morse (12)



NOTE: Curves 2 and 3 show relationship $\frac{K_w}{K} = (\frac{R_o}{R})^2$; Equation (16).

FIGURE 6.

resistivity and saturation is

$$R_0/R = S$$

Consider, in this light, the case of a consolidated sand with more or less stratification. If the planes of stratification are such that flow paths are no longer random, then preferential flow channels will be established either in the general direction of flow or across the general direction of flow. If along the direction of flow the conditions will tend toward those of the capillary tube, and the relation between relative resistivity and saturation will become

$$R_0/R = S^n \qquad \text{where } n < 2$$

If stratification is more pronounced across the direction of electrical flow the conditions will be on the other side of isotropic conditions, and

$$R_0/R = S^n \qquad \text{where } n > 2$$

PROPOSED FUTURE RESEARCH

Since the data presented are somewhat varied and were not gathered with this problem specifically in mind they are not ideally suited for the confirmation of the relationships postulated. It would therefore be desireable to obtain additional data under circumstances controlled to point up the effects considered.

Although it is felt that the fluid distribution within a core, as well as the actual saturation, will effect the resistivity the initial step in the analysis should be to establish this fact experimentally .

Such experimental verification on a macroscopic level could be obtained by placing a partially saturated core in a centrifuge and

EXPERIMENTAL SETUP FOR MEASURING RESISTIVITY AND PERMEABILITY

Vertical Cross Section

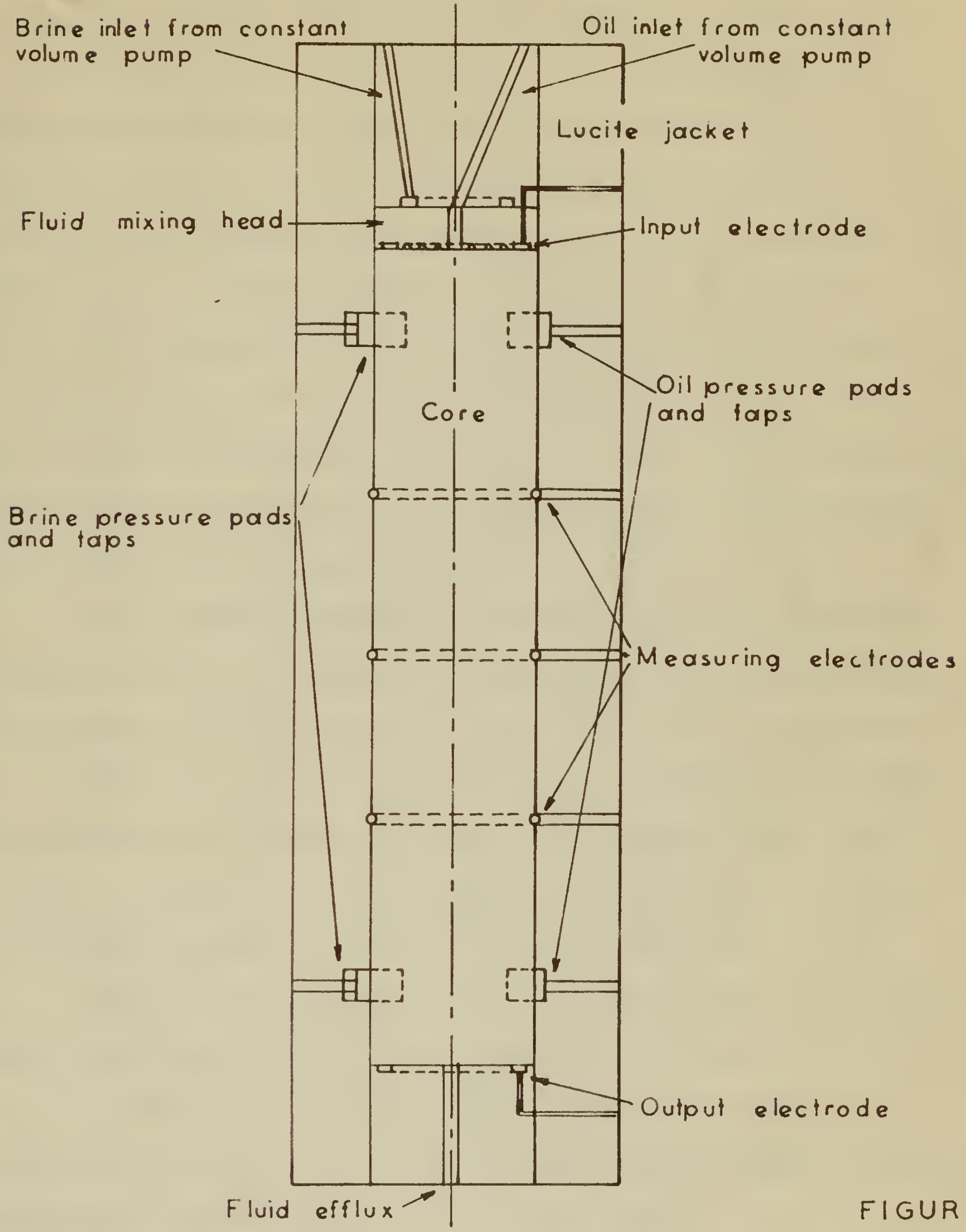

Brine inlet from constant volume pump
Oil inlet from constant volume pump
Lucite jacket
Fluid mixing head
Input electrode
Oil pressure pads and taps
Core
Brine pressure pads and taps
Measuring electrodes
Output electrode
Fluid efflux

FIGURE 7.

Scale: 1" = 1"

rotating the centrifuge at velocities sufficiently high to obtain definite amounts of fluid migration. As the orientations of the core were changed fluid migrations would take place to various parts of the core and fluid distributions would be changed. The rotor of the centrifuge must be fitted with slip rings to measure the resistivity of the core while the centrifuge is rotating. A control plug of known, and constant resistivity must be placed in the centrifuge to keep a check on circuit variations. Such a study would show the effect of changes in the macroscopic fluid distribution within the core. Microscopic variations would probably have an effect upon the resistivity, as indicated in the earlier discussion, but these variations would be masked by the larger macroscopic effect.

Having established such an effect the problem of a relationship is primarily that of the relationship between hydraulic and electric flow. Thus a system in which both hydraulic and electric flow can occur must be used. Further, since there are only two electrical "phases", the conducting fluid and the non-conducting fluid, only a two phase hydraulic system is necessary. Such a system could be set up using a modification of the equipment described by Dunlap(7), shown in Figure (7).

Knowing the homogeneous fluid permeability of the core, and its effective porosity, the core mounted in such a lucite core holder can be saturated with brine. At this saturation the voltage drops can be measured and the resistivity, R_o, can be calculated. The multiple electrodes provide a check on the uniformity of saturation.

The saturation can then be varied by flowing oil and water through the system at rates fixed by the constant volume pumps. When equilibrium is reached the fluid flux out will be equal to the amounts flowing in and a constant pressure drop will obtain. From these rates of flow the permeabilities to both phases can be calculated, although in this analysis only the permeability to the brine phase is required.

While the system is flowing under equilibrium conditions a voltage can be impressed across the end electrodes. The voltage across the measuring electrodes can be measured, and from this the resistivity, R_t, can be calculated. To establish whether or not the resistivity will be effected by streaming potential phenomena, hydraulic flow can be stopped, the voltage drops measured and the resistivity under static conditions calculated. It is considered that for brine concentrations greater than 0.1N this effect will be small.

This then will permit permeability and resistivity measurements to be made at various saturations. In order to make these data definitive of the conditions an independent determination of the brine saturation must be made. This might be accomplished by a mass balance, but, because of the relatively large quantities of fluid involved, the precision of this method will be poor.

A more satisfactory approach to the problem lies in saturation evaluation by X-ray analysis. If the equipment described above were mounted in X-ray equipment as described by Laird and Putnam(16), after each set of permeability and resistivity measurements, the saturation and its distribution could be evaluated independently.

With data obtained in this fashion the reliability of the relationships developed above could be checked conclusively.

CONCLUSIONS

1. It has been shown that because of the similarity between electrical and hydraulic flow a relation may be developed between electrical conductivity or resistivity and hydraulic permeability. This relationship takes the form

$$K_w/K = (R_0/R)S^2$$

2. If the saturation is eliminated from the expression, a second relation results

$$K_w/K = (R_0/R)^{u/(u-2)}$$

(a) Experimental data show that for clean unconsolidated sands $u = 4$, and

$$K_w/K = (R_0/R)^2$$

(b) For consolidated sands the data available are not as conclusive, but a qualitative relation may be written

$$K_w/K = (R_0/R)^v \qquad \text{where } v < 2$$

3. It has also been shown that the relation postulated by Archie

$$S^2 = R_0/R$$

is quite valid for clean unconsolidated sands.

TENTATIVE ADDITIONAL CONCLUSIONS

In addition to the conclusions given above the following tentative conclusions are also presented.

1. For isotropic media the relative permeability to the wetting phase is equal to the square of the relative resistivity. Further, for these media, the relationship developed by Archie is valid and the exponent of the saturation is equal to 2.

2. For anisotropic media, in which the bedding lies in the direction of electrical flow, Archie's relationship is valid, but the exponent of the saturation is less than 2.

3. For anisotropic media in which the bedding is across the direction of electrical flow, Archie's relationship is valid, but the exponent of the saturation is greater than 2.

By a study of the resistivities of consolidated sands in carefully oriented positions these conclusions could be verified, and, if true, a quantitative relationship established. A study of available field data might also be used to investigate these relationships.

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APPENDIX I

EVALUATION OF RELATIVE RESISTIVITY
IN CERTAIN IDEALIZED PORE SPACES

Evaluation of Relative Resistivity in Certain Idealized Pore Spaces

To determine whether or not a definite relationship can be established to predict the variation in resistivity with saturation in any porous matrix, this variation was studied in certain idealized pore spaces. The pore spaces studied in detail were (1) a capillary tube, (2) a spherical pore with a small opening at either end, (3) a pore composed of the frustrums of two right circular cones base to base, and (4) the pore space formed by the cubic packing of identical spheres.

The analysis of each of these pore spaces and the results obtained are presented in detail below.

The symbols used throughout this analysis are:

R, r, r_0, radii of the pores at various cross sections,

R_v, radius of the non-conducting bubble,

Res, resistance of a single pore,

A, area of the rectangular parallelepiped enclosing the pore at right angles to the direction of flow,

L, length of the rectangular parallelepiped enclosing the pore parallel to the direction of flow,

ρ resistivity,

ρ_w conducting fluid resistivity,

S, conducting fluid fractional saturation,

S_n, non-conducting fluid fractional saturation,

c, c', H, g, k, θ G, geometric parameters of the various pores.

Capillary Tube

Consider a capillary tube of dimensions as given by Figure I-1. The resistance of such a pore filled with a brine of resistivity ρ_w may be written directly.

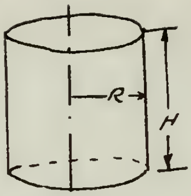


Figure I-1

$$Res_o = \rho_w \frac{H}{\pi R^2} \qquad (1)$$

$$A = 4R^2 \qquad L = H$$

$$\rho_o = Res_o \left(\frac{A}{L}\right) = \rho_w \left(\frac{H}{\pi R^2}\right)\left(\frac{4R^2}{H}\right) = \frac{4\rho_w}{\pi} \qquad (2)$$

In the case of partial saturation if the non-conducting fluid takes the form of a cylinder concentric with, and within the pore, the partially saturated pore takes the configuration given by Figure I-2.

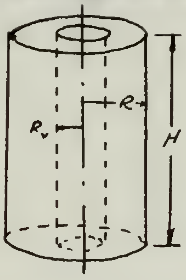


Figure I-2

Assuming that a negligible amount of current flows through the non-conducting fluid, the resistance is:

$$Res_i = \rho_w \frac{H}{\pi(R^2 - R_v^2)} \qquad (3)$$

The fractional saturation of the non-conducting fluid may be written:

$$S_n = \frac{\pi R_v^2 H}{\pi R^2 H} = \left(\frac{R_v}{R}\right)^2$$

And

$$R_v = R S_n^{1/2} \qquad (4)$$

Combining (3) and (4)

$$Res_i = \rho_w \frac{H}{\pi R^2 (1 - S_n)} = \frac{\rho_w H}{\pi R^2 S_i} \qquad (5)$$

Then

$$\rho_i = Res_i \left(\frac{A}{2}\right) = \frac{4 \rho_w H}{\pi S_i} \qquad (6)$$

From (2) and (6) the relationship for the relative resistivity may be written:

$$\frac{\rho_o}{\rho_i} = \frac{\frac{4\rho_w}{\pi}}{\frac{4\rho_w}{\pi S_i}} = S_i \qquad (7)$$

Or in the capillary tube pore the resistivity varies linearly with the conducting fluid saturation.

Spherical Pore

The spherical pore studied is shown by Figure I-3. The initial condition is that where the pore is completely filled with brine.

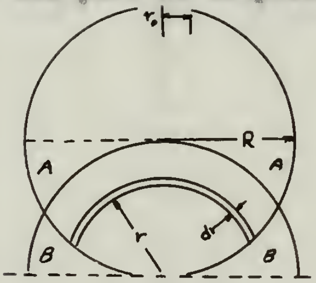


Figure I-3

In determining the resistance of the spherical pore the resistances of incremental spherical shells, varying in radius from r_0 to R, and of thickness dr, are summed up. Since the solid angle of integration will be from 0 to π radians (ie, for hemispheres), the addition of resistances B will be compensated by the omission of resistances A.

$$Res_0 = 2\rho_w \int_{r_0}^{R} \frac{dr}{2\pi r^2} = \frac{\rho_w}{\pi}\left(\frac{1}{r_0} - \frac{1}{R}\right) \qquad (8)$$

$$\rho_0 = Res_0\left(\frac{A}{L}\right) = \frac{\rho_w}{\pi}\left(\frac{1}{r_0} - \frac{1}{R}\right)\left(\frac{4R^2}{2R}\right) = \frac{2\rho_w R}{\pi}\left(\frac{1}{r_0} - \frac{1}{R}\right) \qquad (9)$$

For $r_0 = \frac{R}{c}$

$$\rho_0 = \left(\frac{2\rho_w}{\pi}\right)(c-1)$$

For partial saturation consider a spherical bubble of non-conducting fluid of radius R_v at the center of the pore. (Figure I-4). A somewhat different analysis must be used for this condition. In the region $r_0 < r < R_v$ the spherical shells used for the completely saturated pore are again a valid approach to the summation of resistivities. In the region $R_v < r < R$ flat washerlike discs are used because here they are representative of the flow pattern.

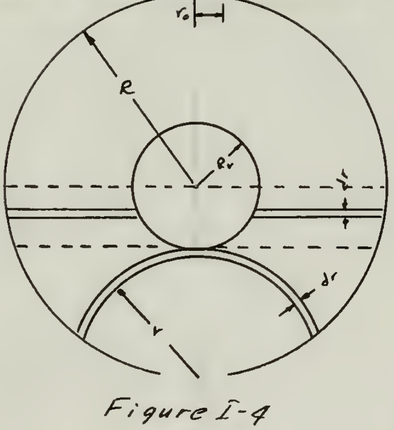


Figure I-4

Thus the resistance at any partial saturation, (i.e.

becomes:

$$Res_i = 2\rho_w \int_{r_o}^{R-R_v} \frac{dr}{2\pi r^2} + 2\rho_w \int_{R-R_v}^{R} \frac{dr}{\pi(R^2-R_v^2)}$$

$$= \frac{\rho_w}{\pi}\left[\left(\frac{1}{r_o} - \frac{1}{R-R_v}\right) + \left(\frac{2R_v}{R^2-R_v^2}\right)\right] \qquad (11)$$

From which:

$$\rho_i = \frac{2\rho_w R}{\pi}\left[\left(\frac{1}{r_o} - \frac{1}{R-R_v}\right) + \left(\frac{2R_v}{R^2-R_v^2}\right)\right] \qquad (12)$$

For $r_o = \frac{R}{c}$

$$\rho_i = \frac{2\rho_w R}{\pi}\left[\left(\frac{c}{R} - \frac{1}{R-R_v}\right) + \left(\frac{2R_v}{R^2-R_v^2}\right)\right] \qquad (13)$$

As before:

$$S_n = \frac{\frac{4}{3}\pi R_v^3}{\frac{4}{3}\pi R^3} = \left(\frac{R_v}{R}\right)^3$$

And

$$R_v = R S_n^{1/3}$$

From (13)

$$\rho_i = \frac{2\rho_w}{\pi}\left[c - \frac{1}{1-S_n^{1/3}} + \frac{2S_n^{1/3}}{1-S_n^{2/3}}\right] \qquad (14)$$

Then the relative resistivity becomes:

$$\frac{\rho_o}{\rho_i} = \frac{\frac{2\rho_w}{\pi}(c-1)}{\frac{2\rho_w}{\pi}\left[c - \frac{1}{1-S_n^{1/3}} + \frac{2S_n^{1/3}}{1-S_n^{2/3}}\right]}$$

$$= 1 - \frac{c(S_n + S_n^{1/3})}{c(1-S_n^{2/3}) + (S_n - S_n^{2/3} + S_n^{1/3} - 1)} \qquad (15)$$

Writing (15) more generally

$$\frac{\rho_o}{\rho_i} = \varphi(c, S) \qquad (16)$$

Conical Pore

Consider a conical pore consisting of two right circular cones base to base. (Figure I-5). In summing up the resistances in such a pore the differential element considered is a spherical cap of thickness dr.

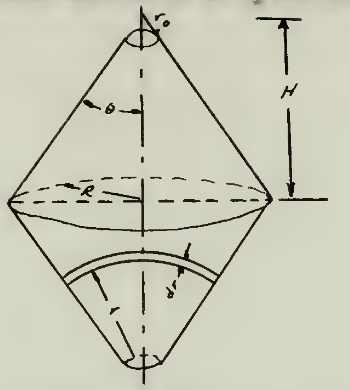


Figure I-5

The resistance of the brine filled pore is given by:

$$Res_0 = 2\rho_w \int_{r_0}^{H} \frac{dr}{4\pi r^2 \sin^2 \frac{\theta}{2}}$$

$$= \frac{\rho_w}{2\pi \sin^2 \frac{\theta}{2}} \left(\frac{1}{r_0} - \frac{1}{H} \right) \qquad (17)$$

Let $r_0 = \frac{R}{c}$; $H = \frac{R}{c'}$

Then

$$\rho_0 = Res_0 \left(\frac{A}{L} \right) = Res_0 \left(\frac{4R^2}{2H} \right) = Res_0 \, (2c'R)$$

And

$$\rho_0 = \frac{c' \rho_w}{\pi \sin^2 \frac{\theta}{2}} (c - c') \qquad (18)$$

For a condition of partial saturation a spherical bubble of radius R_v exists in the center of the pore, and the same treatment as for the partially saturated spherical pore may be used.

In the region $r_0 < r < H - R_v$ elements consisting of spherical caps are used; in the region $H - R_v < r < H$ washer shaped differential elements are used as before. (Figure I-6).

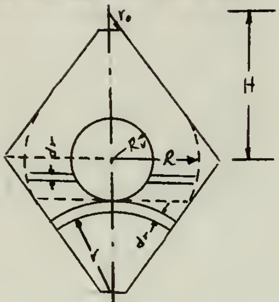


Figure I-6

Further, in the region $H - R_v < r < H$ there will be little electrical flow in the sharp corners where the bases join, and hence a spherical outline, indicated by the dotted line Figure I-6, of radius R is used.

Then:

$$Res_i = 2\rho_w \int_{r_0}^{H-R_v} \frac{dr}{4\pi r^2 \sin^2\frac{\theta}{2}} + 2\rho_w \int_{H-R_v}^{H} \frac{dr}{\pi(R^2 - R_v^2)}$$

$$= \frac{\rho_w}{2\pi \sin^2\frac{\theta}{2}}\left(\frac{1}{r_0} - \frac{1}{H-R_v}\right) + \frac{2\rho_w}{\pi}\left(\frac{R_v}{R^2 - R_v^2}\right) \qquad (19)$$

$$\rho_i = \frac{\rho_w c'}{\pi \sin^2\frac{\theta}{2}}\left[(R)\left(\frac{1}{r_0} - \frac{1}{H-R_v}\right)\right] + \frac{4\rho_w c'}{\pi}\left[(R)\left(\frac{R_v}{R^2 - R_v^2}\right)\right] \qquad (20)$$

Let $r_0 = \frac{R}{c}$; $H = \frac{R}{c'}$

As in the previous cases

$$S_n = \frac{\frac{4}{3}\pi R_v^3}{\frac{2}{3}\pi R^3\left(\frac{c^2+c+1}{c'c^2}\right)} = \frac{2R_v^3}{R^3 g}$$

where $g = \frac{c^2+c+1}{c'c^2}$

And $R_v = S_n^{1/3}\left(\frac{g}{2}\right)^{1/3} R = R k S_n^{1/3}$

where $k = \left(\frac{g}{2}\right)^{1/3}$

Rewriting equation (20)

$$\rho_i = \frac{\rho_w c'}{\pi \sin^2 \frac{\theta}{2}} \left[c - \frac{c'}{1 - S_n^{1/3} c' k} \right] + \frac{4 \rho_w c'}{\pi} \left[\frac{S_n^{1/3} k}{1 - S_n^{2/3} k^2} \right] \qquad (21)$$

Then the relative resistivity becomes:

$$\frac{\rho_o}{\rho_i} = \frac{\dfrac{\rho_w c'}{\pi \sin^2 \frac{\theta}{2}} (c - c')}{\dfrac{\rho_w c'}{\pi \sin^2 \frac{\theta}{2}} \left(c - \dfrac{c'}{1 - S_n^{1/3} c' k} \right) + \dfrac{4 c' \rho_w}{\pi} \left(\dfrac{S_n^{1/3} k}{1 - S_n^{2/3} k^2} \right)}$$

$$= \frac{1}{1 + \dfrac{S_n^{1/3} k}{c - c'} \left(\dfrac{c'^2}{1 - S_n^{1/3} c' k} + \dfrac{4 \sin^2 \frac{\theta}{2}}{1 - S_n^{2/3} k^2} \right)} \qquad (22)$$

Since c, c', k and θ, are all dependent upon the geometry of the pore they may be lumped together in a single geometric parameter G. Doing this a generalized relation for the relative resistivity may be written. As before this relationship shows that the resistivity is dependent upon the saturation and the geometrical configuration. Thus the relative resistivity is:

$$\frac{\rho_o}{\rho_i} = \varphi(G, S) \qquad (23)$$

Cubic Packing of Uniform Spheres

The pore spaces formed by the cubic packing of uniform spheres are regular, but somewhat more complex than those previously discussed, resembling in cross section a hypocycloid of four cusps. Because the

density of the electrical flow field is distributed regularily, the resistances may be summed up, in the case of the completely brine filled pore, in terms of flat discs of radius r and thickness dr. For the partially saturated core in the region of the non-conducting bubble, washer shaped elements must be used. Figures I-7 and I-8 represent such a pore in a fully and partially saturated condition respectively.

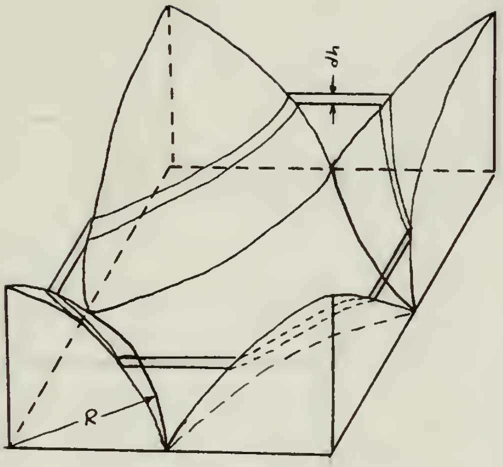


Figure I-7

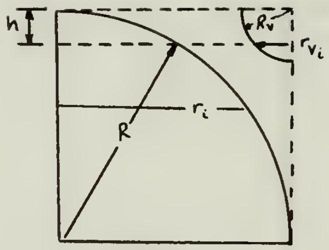


Figure I-8

The area of the opening at the entrance to the pore is:

$$a_e = 4R^2 - \pi R^2 = R^2(4-\pi)$$

The area at the center of the pore is:

$$a_c = 4R^2$$

And any intermediate area may be written:

$$a_i = R^2(4-\pi) + \pi h^2$$

Then the resistance of a single pore is:

$$Res_0 = 2P_w \int_0^R \frac{dh}{(4-\pi)R^2 + \pi h^2}$$

$$= \frac{2P_w}{\pi R \sqrt{\frac{4}{\pi}-1}} \tan^{-1} \frac{1}{\sqrt{\frac{4}{\pi}-1}} \qquad (24)$$

From which:

$$[illegible] = \left(\frac{4\rho_w}{\pi}\right)\left(\frac{1}{\sqrt{\frac{4}{\pi}-1}}\tan^{-1}\frac{1}{\sqrt{\frac{4}{\pi}-1}}\right) \qquad (25)$$

For the condition of partial saturation, the area available to flow in the region of the non-conducting bubble depends upon the size of the bubble, and may be written:

$$A_{v_i} = 4R^2 - \pi\left[2Rh - 2h^2 + R_v^2\right]$$

The area available to flow outside this region is:

$$A_i = 4R^2 - 2\pi Rh + \pi h^2$$

Then the resistance of the partially saturated core becomes:

$$Res_i = 2\rho_w\int_0^{R_v}\frac{dh}{4R^2-\pi R^2-2\pi Rh+2\pi h^2} + 2\rho_w\int_{R_v}^{R}\frac{dh}{4R^2-2\pi Rh+\pi h^2}$$

$$= \frac{2\rho_w}{\pi}\left\{\frac{1}{\sqrt{R^2(\frac{8}{\pi}-1)-2R_v^2}}\left[\tan^{-1}\frac{2R_v-R}{\sqrt{R^2(\frac{8}{\pi}-1)-2R_v^2}} + \tan^{-1}\frac{R}{\sqrt{R^2(\frac{8}{\pi}-1)-2R_v^2}}\right]\right\}$$

$$+ \frac{2\rho_w}{\pi R}\left\{\frac{1}{\sqrt{\frac{4}{\pi}-1}}\tan^{-1}\frac{R-R_v}{R\sqrt{\frac{4}{\pi}-1}}\right\} \qquad (26)$$

Evaluating R_v in terms of R and S_n

$$R_v = R\left[S_n\left(\frac{6}{\pi}-1\right)\right]^{1/3}$$

Let $\left[S_n\left(\frac{6}{\pi}-1\right)\right]^{1/3} = \sigma$

Then equation (26) becomes:

$$Res_i = \frac{2\rho_w}{\pi R}\left\{\frac{1}{\sqrt{\frac{8}{\pi}-1-2\sigma^2}}\left[\tan^{-1}\frac{2\sigma-1}{\sqrt{\frac{8}{\pi}-1-2\sigma^2}} + \tan^{-1}\frac{1}{\sqrt{\frac{8}{\pi}-1-2\sigma^2}}\right.\right.$$

$$\left.+ \frac{1}{\sqrt{\frac{4}{\pi}-1}}\tan^{-1}\frac{1-\sigma}{\sqrt{\frac{4}{\pi}-1}}\right\} \qquad (27)$$

From which:

$$\rho_i = \frac{4\rho_w}{\pi}\left\{ \frac{1}{\sqrt{\frac{8}{\pi}-1-2\sigma^2}} \tan^{-1} \frac{\sigma\sqrt{\frac{8}{\pi}-1-2\sigma^2}}{\frac{4}{\pi}-\sigma^2-\sigma} + \frac{1}{\sqrt{\frac{4}{\pi}-1}} \tan^{-1} \frac{1-\sigma}{\sqrt{\frac{4}{\pi}-1}} \right\} \qquad (28)$$

Then the relative resistivity becomes:

$$\frac{\rho_o}{\rho_i} = \frac{\frac{1}{\sqrt{\frac{4}{\pi}-1}} \tan^{-1} \frac{1}{\sqrt{\frac{4}{\pi}-1}}}{\frac{1}{\sqrt{\frac{8}{\pi}-1-2\sigma^2}} \tan^{-1} \frac{\sigma\sqrt{\frac{8}{\pi}-1-2\sigma^2}}{\frac{4}{\pi}-\sigma^2-\sigma} + \frac{1}{\sqrt{\frac{4}{\pi}-1}} \tan^{-1} \frac{1-\sigma}{\sqrt{\frac{4}{\pi}-1}}} \qquad (29)$$

Or writing (29) more generally

$$\frac{\rho_o}{\rho_i} = \varphi(G, S) \qquad (30)$$

From the development given above it may be seen that although the relative resistivity will in general take the form

$$\frac{\rho_o}{\rho_i} = \varphi(G, S)$$

there is no simple precise relationship between saturation and resistivity. As the system begins to approach a natural system, in the cubic packing of spheres, the relationship becomes very formidable indeed.

For this reason the problem was studied in terms of a additional parameter, that of the variation of hydraulic permeability with saturation.

If the equations for $\frac{\rho_o}{\rho_i}$ are examined in the region where S_n is small, the following approximate relations may be arrived at by neglecting the higher powers of S_n.

For the spherical pore:

$$\frac{\rho_o}{\rho_i} \approx 1 - S_n^{1/3} \qquad (31)$$

For the conical pore:

$$\frac{\rho_o}{\rho_i} \approx 1 - \frac{S_n}{e}^{1/3} \qquad (32)$$

For the pore formed by the cubic packing of uniform spheres:

$$\frac{\rho_o}{\rho_i} \approx 1 - 1.1\, S_n^{1/3} \qquad (33)$$

To establish the significance of the higher order terms in these relationships, equation (29) was plotted on logarithmic coordinates along with equation (33). The experimental data of Figure 1. is also shown for comparison purposes. It may be seen immediately that the higher order terms are significant all the way and such a simplification is a poor approximation.

It should be noted, however, that the line of the experimental data is somewhat an average of these two curves. This is not surprizing when one considers that the ideal pore shapes were studied as single pores, while in a randomly packed porous medium many of the individual effects will cancel each other.

Since such an averagingprocess will effect the electrical conductivity and hydraulic permeability similarily a relationship between the two would tend to cancel out the random effects and make the problem soluble.

THEORETICAL RELATIVE RESISTIVITY
CUBIC PACKING OF SPHERES

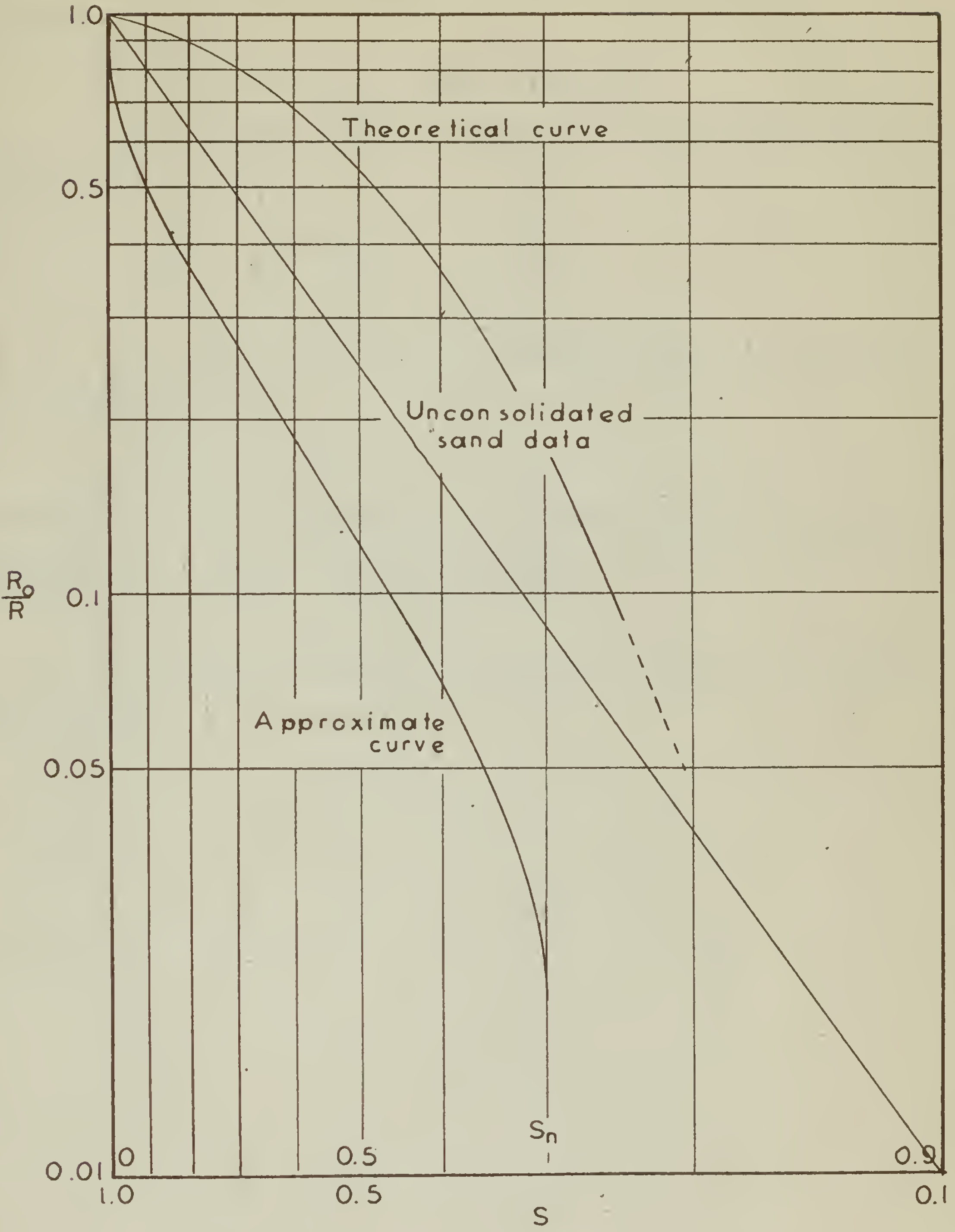


FIGURE 9.

Conclusions

From this analysis it may be concluded:

1. The relationship between saturation and resistivity will, in general, take the form

$$\frac{\rho_o}{\rho_i} = \varphi(G, S)$$

2. Approximate relations neglecting higher powers of S_n, may be shown to take the form

$$\frac{\rho_o}{\rho_i} \simeq 1 - c S_n^{1/3}$$

However these are poor approximations because the higher order terms are significant throughout the entire range.

3. An additional independent parameter is required for solution of the problem.

APPENDIX II

DATA

Relative Resistivity and Relative Permeability for Unconsolidated Sands

Table I

(Wyckoff & Botset)(11)

S	R_o/R	$(R_o/R)^2$	K_w/K
1.0	1.0	1.0	1.0
0.9	0.84	0.702	0.70
0.8	0.67	0.45	0.46
0.7	0.53	0.28	0.29
0.6	0.39	0.15	0.16
0.5	0.27	0.073	0.08
0.4	0.17	0.029	0.03
0.3	0.09	0.008	0.01
0.2	0.04	0.0016	0.005

Table II

(Leverett)(14)

S	R_o/R	$(R_o/R)^2$	K_w/K
1.0	1.0	1.0	1.0
0.9	0.845	0.712	0.74
0.8	0.69	0.475	0.53
0.7	0.54	0.29	0.36
0.6	0.40	0.16	0.22
0.5	0.27	0.073	0.11
0.4	0.17	0.029	0.05
0.3	0.08	0.006	0.01
0.2	0.04	0.0016	-

Table III

(Kogan)(15)

S	R_o/R	$(R_o/R)^2$
1.0	1.0	1.0
0.9	0.81	0.65
0.8	0.67	0.45
0.7	0.54	0.29
0.6	0.41	0.17
0.5	0.29	0.084
0.4	0.20	0.040
0.3	0.12	0.014
0.2	0.06	0.004

Table IV

(Dunlap)(7)

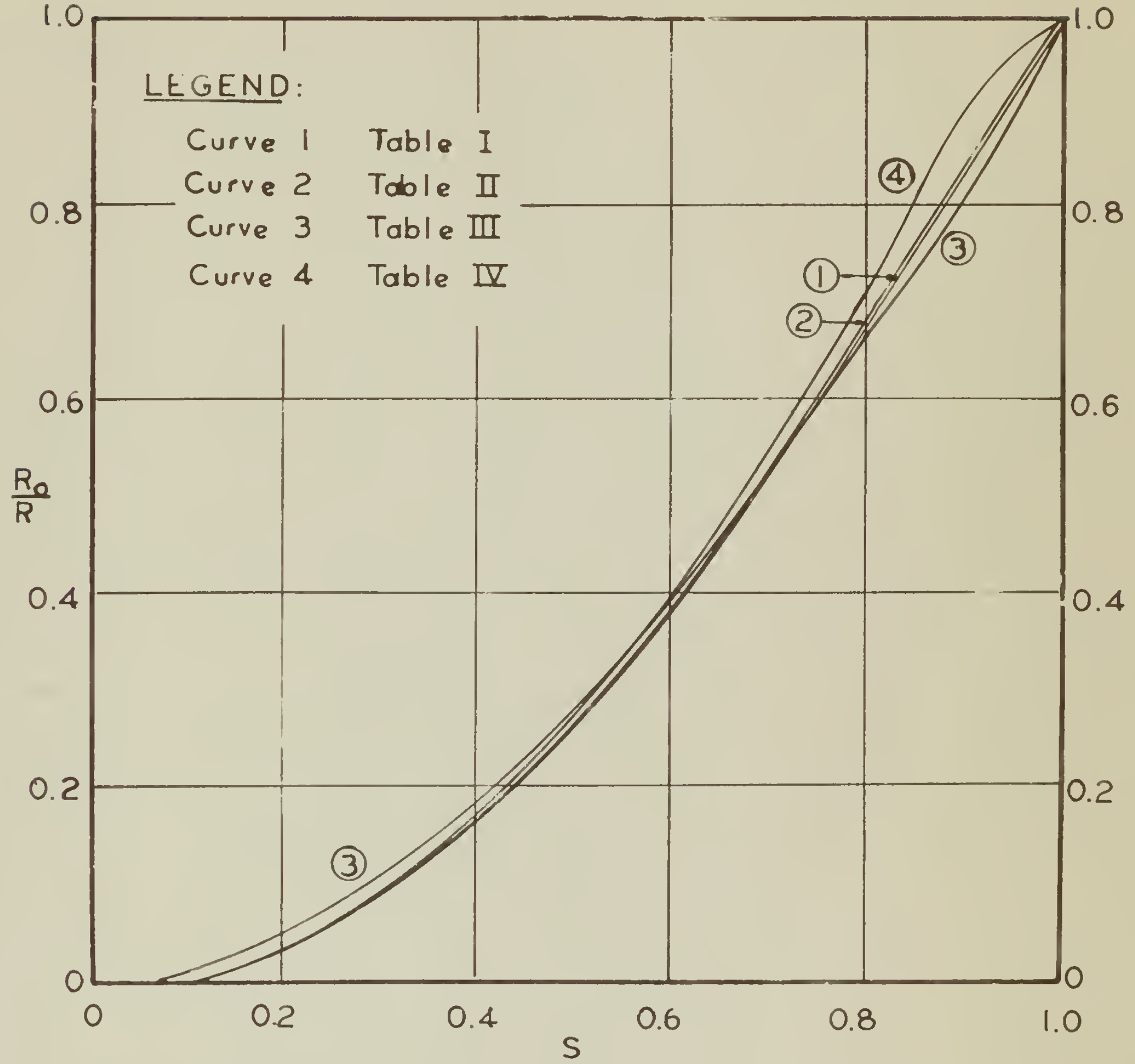
S	R_o/R	$(R_o/R)^2$
1.0	1.0	1.0
0.9	0.92	0.845
0.8	0.72	0.52
0.7	0.54	0.29
0.6	0.39	0.15
0.5	0.28	0.08
0.4	0.17	0.029
0.3	0.08	0.006
0.2	0.04	0.002

Table V

(Average)

S	R_o/R	$(R_o/R)^2$	K_w/K
1.0	1.0	1.0	1.0
0.9	0.854	0.729	0.72
0.8	0.688	0.473	0.495
0.7	0.538	0.279	0.325
0.6	0.398	0.158	0.190
0.5	0.278	0.077	0.095
0.4	0.180	0.032	0.040
0.3	0.098	0.0096	0.01
0.2	0.045	0.002	-

SPREAD OF RELATIVE RESISTIVITY DATA FOR UNCONSOLIDATED SANDS



NOTE: The average of these curves is represented by curve 1, Figure 4.

FIGURE 8.

Relative Resistivity and Relative Permeability Data

Table VI

(Consolidated Natural Core)

(Morse)(12)

Bradford Sand, Air Permeability 93 md., Porosity 20.2%

S	R_o/R	$(R_o/R)^2$	$(R_o/R)^{1.5}$	K_w/K
1.0	1.0	1.0	1.0	1.0
0.9	0.40	0.150	0.253	0.37
0.8	0.25	0.063	0.126	0.24
0.7	0.17	0.029	0.071	0.12
0.6	0.11	0.012	0.037	0.05
0.5	0.07	0.005	0.018	0.01
0.4	0.05	0.002	0.010	-

Table VII

(Consolidated Synthetic Core)

(Morse)(12)

S	R_o/R	$(R_o/R)^2$	K_w/K
1.0	1.0	1.0	1.0
0.9	0.84	0.71	-
0.8	0.73	0.53	0.53
0.7	0.54	0.29	0.31
0.6	0.42	0.18	0.18
0.5	0.31	0.11	0.11
0.4	0.20	0.04	0.05
0.3	0.12	0.014	0.015

Relative Resistivity Data

for Clean Unconsolidated Sand

Table VIII

(Dunlap(7), Figure 5)

Top Part of Core			Middle Part of Core			Bottom Part of Core		
S	R_o/R	$(R_o/R)S^2$	S	R_o/R	$(R_o/R)S^2$	S	R_o/R	$(R_o/R)S^2$
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.89	0.917	0.725	0.74	0.86	0.470	0.80	1.0	0.640
0.82	0.735	0.526	0.67	0.307	0.138	0.73	0.833	0.444
0.75	0.550	0.319	0.58	0.153	0.052	0.67	0.833	0.374
0.65	0.306	0.129	0.53	0.116	0.033	0.57	0.714	0.232
0.58	0.153	0.052	0.47	0.100	0.0220	0.52	0.312	0.084
0.54	0.121	0.035	0.44	0.092	0.0178	0.50	0.278	0.0695
0.49	0.112	0.027	0.42	0.075	0.0132	0.45	0.212	0.0428
0.45	0.106	0.021	0.36	0.070	0.0091	0.42	0.200	0.0353
0.43	0.097	0.018	0.32	0.063	0.0065	0.37	0.143	0.0196
0.37	0.087	0.012	0.30	0.061	0.0055	0.32	0.125	0.0128
0.33	0.078	0.0085	0.29	0.061	0.0051	0.27	0.100	0.0073
0.30	0.078	0.0070	0.27	0.061	0.0044	0.25	0.077	0.0048
0.27	0.073	0.0056	0.24	0.054	0.0031	0.24	0.062	0.0036
0.24	0.073	0.0042	0.23	0.048	0.0025	0.22	0.049	0.00216
0.23	0.061	0.0032	0.21	0.041	0.00180	0.20	0.043	0.00172
0.22	0.054	0.0026	0.20	0.037	0.00148	0.19	0.041	0.00148
0.21	0.050	0.00220	0.18	0.029	0.00094	0.17	0.031	0.00090
0.19	0.048	0.00173	0.17	0.025	0.00072	0.15	0.020	0.00045
0.18	0.042	0.00136	0.15	0.018	0.00040	0.14	0.019	0.00037
0.17	0.032	0.00092	0.13	0.014	0.00024	0.13	0.014	0.00024

Relative Permeability Data for Unconsolidated Sands

Table IX

(Wyckoff & Botset)(11)

S	K_w/K
1.0	1.0
0.9	0.70
0.8	0.46
0.7	0.29
0.6	0.16
0.5	0.08
0.4	0.035
0.3	0.010
0.2	0.005

Relative Resistivity and Relative Permeability Data for a Consolidated Sand

Table X

(Morse)(12)

Bradford Sand, Air Permeability 93 md., Porosity 20.2%

S	R_o/R	$(R_o/R)S^2$	K_w/K
1.0	1.0	1.0	1.0
0.9	0.40	0.324	0.37
0.8	0.25	0.160	0.24
0.7	0.17	0.083	0.12
0.6	0.11	0.0396	0.04
0.5	0.07	0.0175	0.01
0.4	0.05	0.0080	–

APPENDIX III

DEFINITION OF "RELATIVE SENSITIVITY"

[illegible]

"[illegible]" [illegible]

Definition of "Relative Resistivity"

Relative resistivity as used throughout this work is actually the Relative reciprocal resistivity, or the relative conductivity. However for clarity and brevity, the term relative resistivity is used.

It should be noted that relative reciprocal resistivity, or relative conductivity, is the electrical analog of relative permeability and varies between 1.0 and 0.0. Relative resistivity on the other hand varies between 1.0 and infinity. It is the former term, with appropriate limits, which is used throughout this work.

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